

Finite Element Analysis of Granular Pavements Considering Material Nonlinearity

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Abstract: Empirical approach is in use for the designing of granular or low volume pavements in many countries, including India. These types of pavements mainly comprise of unbound granular material layer over subgrade with thin asphalt surfacing. Accurate modeling of the granular material is essential for better correlation of the identified mechanistic responses with the performance of pavements. It is known that, unbound materials are nonlinear and have stress dependent resilient modulus. Hence, the stress dependency of unbound materials should be considered for an accurate estimation of true pavement responses. Writing suitable codes in Finite Element (FE) analysis software, model the nonlinearity of unbound granular layers. A 3-D FE was developed and analyzed. It was observed that the pavement responses obtained from the 3-D FE analysis carried out for typical low volume pavements taking into account the nonlinear characteristics of unbound pavement materials differ by 34% to 44% from those obtained using linear analysis.

Keywords: Low Volume Pavement, Finite Element Method, Pavement Response, Unbound Granular Layer, Nonlinearity.

1. INTRODUCTION

Low volume roads form a major part of the Indian road network, comprising 80% of the total road length. These roads are usually constructed as granular pavements with or without thin asphalt surfacing layer and the design of these pavements is done using an empirical approach. For such type of granular pavements with thin surfacing, granular material constitutes the main structural layer for carrying the traffic load and hence, plays a significant role in their performance. Unbound materials behave nonlinearly under the load application and have stress dependent resilient modulus (Gupta *et al.*, 2014a; Gupta *et al.*, 2015a). Hence, the stress dependency of unbound materials should be considered for an accurate estimation of true pavement responses. Therefore, accurate modeling of the granular material is essential for better correlation of the identified mechanistic responses with the performance of pavements (Gupta *et al.*, 2014c; Gupta *et al.*, 2015b). The main objective of this study is to examine and compare the critical pavement responses obtained from the FE analysis of flexible pavement with linear and nonlinear properties of granular and subgrade layer. Three dimension (3-D) FE models have been developed in this study for the analysis of granular pavements. Various finite element programs are available with high speed computing facilities and this is attracting the present researchers to analyze the complex and typical behavior of pavement structure. ANSYS (ANSYS, 2011) has been used in the present study to develop the 3-D model of granular pavement structure and to incorporate the effect of nonlinearity of materials.

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Macros (i.e., finite element code in ANSYS) were written, which assigns the moduli values to individual elements based on stress level until convergence.

2. FINITE ELEMENT MODELING

There are three approaches that can be used to compute the stresses and strains in pavement structures: layered elastic methods, two-dimensional (2-D) FE modeling, and three-dimensional (3-D) FE modeling. In layered elastic method, the system is divided into an arbitrary number of horizontal layers (Vokas and Robert, 1985). The thickness of each individual layer and material properties may vary from one layer to the next, but in any one layer the material is assumed to be homogeneous and linearly elastic. The 2-D FE analysis assumes plane-strain or axi-symmetric conditions. Plane-strain models cannot accurately reproduce actual traffic loadings. The second FE formulation in use is an axi-symmetric modeling approach. This approach assumes that the pavement structure has constant properties in horizontal planes, and the traffic loading can be modeled as a circular load. This model creates a serious limitation when a dual tire configuration needs to be studied. To overcome the limitations inherent in 2-D modeling approaches, 3-D FE models are becoming more widespread. It can represent the pavement configuration accurately, including loading position, shoulders, and discontinuities such as joints or cracks, by introducing special elements such as springs or pressure elements. With 3-D FE analysis, the effect of non-linear materials or the effect of combination of loads, including un-symmetric or different loading types can be captured (Gupta *et al.*, 2014b). Therefore, a 3-D model has been adopted for analysis in the present study.

2.1 Type of Analysis

Most low volume pavements are primarily thin flexible pavements with an unbound base and subgrades. This is especially true for the behavior of unbound pavement materials, which is nonlinear and stress dependent, even at low traffic stresses. When a wheel load is applied, the unbound base layer spreads wheel load over subgrades. The response of unbound materials in a pavement depends on its stress history and the current stress state (Gupta *et al.*, 2011). A need, therefore, exists for more realistic prediction of pavement response for such pavements, based on proper constitutive models and computational methods. This can be done by developing the nonlinear, stress dependent FE program for pavement analysis. Stress dependent models for the resilient modulus and Poisson's ratio of unbound pavement materials are incorporated into the FE model to predict the resilient behavior within the pavement layers under specified wheel loads. Evaluating the effect of base nonlinearity is especially important here, and relatively few 3-D FE models have considered it. To capture the effects of the material nonlinearity, two cases were considered in this study, as listed below:

- i. Modeling nonlinearity in granular layer with subgrade soil behaving linearly, and
- ii. Modeling nonlinearity in both i.e. subgrade and granular layer.

2.2 Model Geometry

Low volume pavements in India are generally, single lane roads of 3.75 m carriageway width with shoulders on both the sides of the carriageway. Khabiri and Karagaran (2008) performed FE analysis on country roads and reported that shoulder width had minimal effect on stresses

and strains at the top of the subgrade and at the bottom of the base layer. Going by the mentioned finding, the shoulder width was taken as 1500 mm. This is shown in Fig. 1. A length of 1500 mm (Bodhinayake and Hadi, 2004) to 2000 mm (Mulungye *et al.*, 2006) was generally taken in the longitudinal direction for the 3-D modeling of the pavement geometry. Hence, a length of 1800 mm was taken in the longitudinal direction. Due to symmetry in longitudinal and transverse directions, a quarter model was considered. Granular layer thickness of 400 mm was taken in the development of FE model. The depth of subgrade considered in the literature for FE modeling varied from 1500 mm (Werkmeister *et al.*, 2004) to infinite thickness (Harichandran *et al.*, 1989; Cho *et al.*, 2002). Wang (2001) and Mulungye *et al.* (2006) adopted a subgrade depth of 2000 mm, whereas Hadi and Bodhinayake (2003), Kuo and Huang (2006) and Saad *et al.* (2005) considered subgrade depths of 2200, 2300 and 2500 mm respectively. A subgrade depth of 2500 mm was considered in the present study. Sensitivity analysis was also conducted to find the influence of the model geometry, and it suggests that a subgrade depth of 2500 mm is appropriate and hence was adopted. Figure 2 shows the typical two-layer pavement considered for sensitivity analysis.

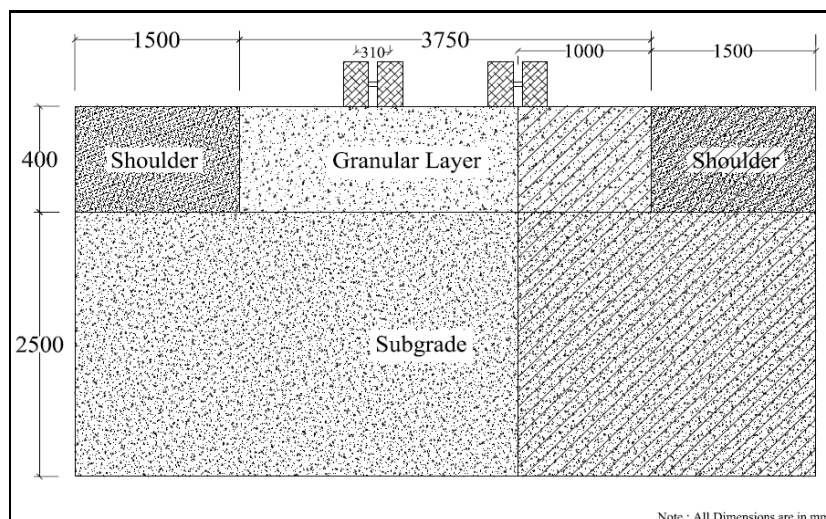


Figure 1. Load placement and pavement section considered for FE analysis

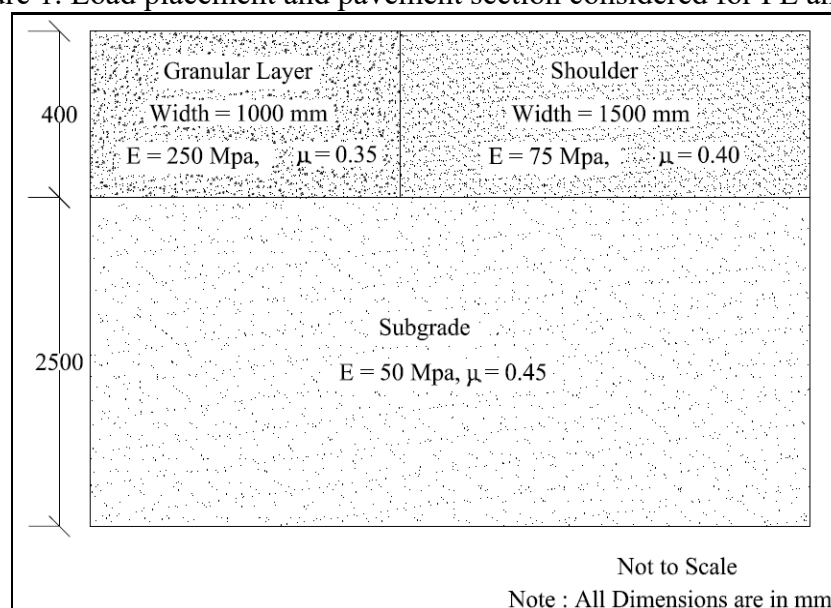


Figure 2. Typical two-layer pavement considered for sensitivity analysis

2.3 Element Type and Boundary Conditions

SOLID-45 element was selected for developing the 3-D FE model in ANSYS environment. The nodes on each face except the surface are constrained from moving along the direction perpendicular to that face. This means that lateral displacement's u_x and u_y are restrained in X and Y directions respectively along the axis of symmetry ($u_x = 0$ and $u_y = 0$).

2.4 Loading Conditions

The load was assumed to be transmitted to the pavement through rectangular contact area of wheels at uniform vertical contact pressure. No horizontal surface shear stresses were considered. The center to center distance between dual wheels was taken as 310 mm in the present study as suggested by Sunkavalli *et al.* (2008). Sunkavalli (2007) measured the dimensions of tire imprints for various combinations of load and tyre pressure. It was found that the width of imprints varies between 190 mm and 200 mm. Based on this study, a wheel contact width of 200 mm was adopted. To simulate the wheel load (20 kN) (Sunkavalli *et al.*, 2008) of a standard axle with a tire pressure of 0.56 MPa (IRC:81, 1997), length of contact area was computed and it came out to be 180 mm. Therefore, an element of size 200×180 mm was considered for the FE model.

2.5 FE Model Validation

Validation of the FE model was done to check the results (in terms of stress and strains) given by it with linear elastic theory results. KENPAVE, a linear elastic layered analysis program developed by Huang (2004) was used to validate the FE model. A typical two layer pavement system as shown in Fig. 2 was analyzed using the FE model considering both the layers to be linearly elastic. Similar pavement was analyzed using KENPAVE also. Circular contact area was considered for the analysis using KENPAVE. Sunkavalli *et al.* (2008) had given the radius of circular contact area with respect to different tire pressures and single wheel load. Corresponding to single wheel load of 20 kN and tire pressure 0.56 MPa, radius of circular contact area was given as 108 mm. For FE model rectangular contact shape with dimension of $200 \text{ mm} \times 180 \text{ mm}$ was used. Base thickness, elastic modulus and Poisson's ratio values were taken to be the same in both the models. In the case of FE model, part of pavement with shoulder was considered, whereas pavement layers were assumed to be infinite in the horizontal direction, in the case of linear elastic layered analysis. Comparison of the results obtained using the FE model and KENPAVE for a linear elastic layered system is shown in Fig. 3. The results of the two analyses were found matching well. Hence, developed FE model was considered fit for carrying out FE analysis of pavement structure as modeled.

3. GRANULAR UNBOUND LAYER NONLINEARITY

In many of the analytical models they consider granular layer to behave linearly elastic i.e. the properties of material remain constant with the application of load. This approach is simple and easy to use but does not represent the actual behavior of the material present in the granular layer i.e. the nonlinear behavior of granular layers. In reality, granular materials are not linearly elastic. The modulus values of the granular material changes with the application of bulk stresses. Also, linear elastic analysis of pavements with thick granular layer (with no surfacing or with thin bituminous surfacing) leads to non-existent high tensile stresses within

the granular layers (Gonzalez *et al.*, 2007). Modeling the nonlinearity of granular material and subgrade soil was done by considering the plastic behavior of unbound materials by most of the general purpose finite element software packages. Zaghoul and White (1993), Hossain and Wu (2002), Saleh *et al.* (2003) and Suleiman and Varma (2007) used Drucker-Prager (DP) plasticity model to represent nonlinear behavior in granular layers and cohesive soils. Kim (2000) used DP and hypo-elastic models for granular material in a 3-D FE model and compared the responses with those obtained using linear elastic analysis. A difference of 10 to 15% was observed in the pavement responses in the case of DP model and 60% in the case of hypo-elastic model. Gonzalez *et al.* (2007) carried out an analysis of a thin sealed granular pavement using a 3-D FE model. They used different nonlinear models such as $k-\theta$ model, universal model and modified universal model. The granular pavements were also analyzed using linear elastic (FE) approach. The analytical responses were compared with the responses obtained from instrumented test track. Results of the nonlinear analyses were in much better agreement with the measured responses compared to those obtained using linear analysis. They found little difference between the responses obtained from different nonlinear models.

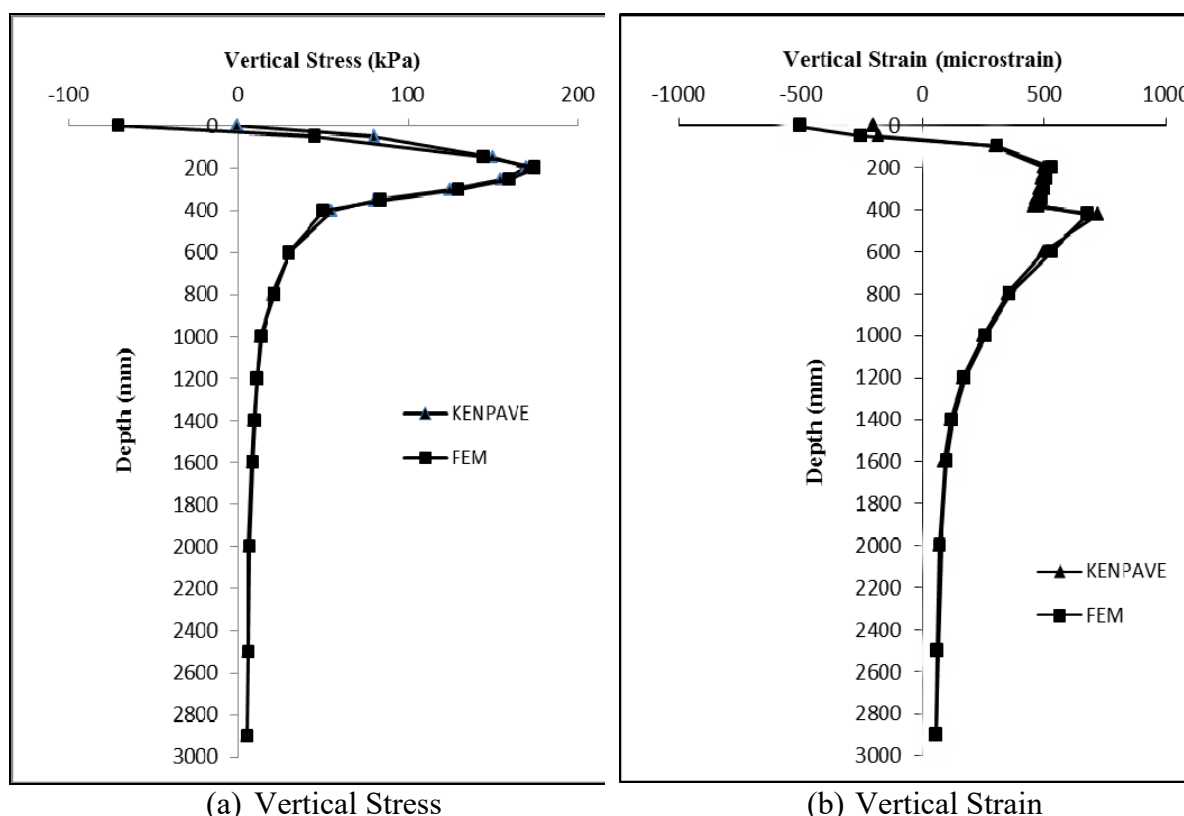


Figure 3. Comparison of results obtained using FE model (linear) and KENPAVE

3.1 Drucker-Prager Model

The Drucker-Prager yield criterion (Drucker and Prager, 1952) is a pressure-dependent model for determining whether a material has failed or undergone plastic yielding. To use the Drucker-Prager model in FE analysis, the input parameters required are the cohesion (c), angle of internal friction (ϕ) and the dilation angle (ψ) along with the linear elastic parameters like elastic modulus and Poisson's ratio. The c , ϕ and ψ values obtained by Morgan (1972) from triaxial shear tests for good quality aggregates were used in this analysis. These were 38

kPa, 53° and 0° respectively. The pavement system was analyzed using the FE model developed in this study. The analysis was carried out by considering the granular layer either linear or nonlinear. Surface deflections computed at different radial distances using linear elastic analyses (from KENPAVE program) were compared with those obtained using nonlinear analysis (DP model). Nonlinear analysis resulted in an increase of about 7% in the central deflection value and 12% increase in vertical strain on the subgrade as compared to the deflection values obtained using linear elastic analysis. This indicated that consideration of nonlinearity affects the critical responses in the system. The comparison is shown in Fig. 4 and Fig. 5 respectively.

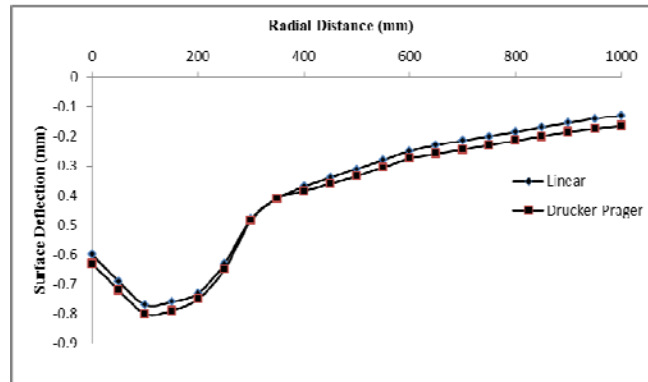


Figure 4. Deflection response from the linear and DP model

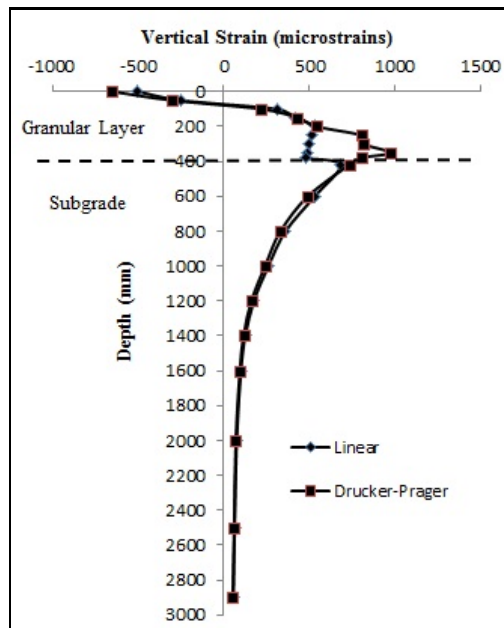


Figure 5. Vertical strain response from linear and DP model

3.2 k-θ Model

k-θ model is a simple model, which requires two material constants. Pandey and Naidu (1994) developed a relationship between resilient modulus and bulk stress. This is given by Eq. 1 and was used in the present study.

$$M_R = 3.47(\theta)^{0.7375} \quad (1)$$

where,

M_R : Resilient modulus in MPa, and
 θ : Bulk stress in kPa.

3.3 Modeling Nonlinearity in ANSYS

As ANSYS is a general purpose finite element code, it does not include specific stress dependent nonlinear elastic models for granular materials. Macros were written to assign material properties for individual elements and compute the elastic modulus (using Eq. 1) for each element in the granular layer depending on the stress condition. A seed modulus was initially assigned to all the elements of the layer for computing the bulk stresses for each element. Geostatic pressure due to self-weight of pavement layers was considered in the model for estimating the bulk stress. Since this is an iterative approach, the material properties change at each iteration according to the stress state of each element until the selected convergence criterion is met. The convergence criterion used by Sahoo and Reddy (2010) was used in the present study. It is given by Eq. 2.

$$\frac{\sum_{i=1}^N abs(E_{new_i} - E_{old_i})}{\sum_{i=1}^N E_{old_i}} \times 100 \leq 5 \quad (2)$$

where,

N : Number of elements in the granular layer
 E_{new_i} : Elastic modulus of i th element for a given iteration, and
 E_{old_i} : Elastic modulus of i th element in the previous iteration.

3.3.1 No Tension Material

There is a potential difficulty with the integration point method. Granular material does not possess any significant tensile strength. To model the material as a no-tension material, negative (i.e. tensile) minor principal stresses are set to zero. After nullifying the tensile stresses, if the computed modulus value becomes too small or nearly zero then it causes convergence problem for iterative solvers like ANSYS. It is due to its sensitivity to poor conditioning of the system stiffness matrix, 'k'. To avoid this, a minimum value, needs to be set for resilient modulus. If the computed modulus value is found to be smaller than a specified minimum modulus value, that particular element is assigned the minimum modulus value and the iteration stops there (Sahoo, 2009). This is an important parameter which can affect likely results. Brown and Pappin (1981) used a minimum value of 100 MPa in SENOL program (FE program) for the elements experiencing tensile stresses. Bose (1994) suggested a value of 80 MPa to 140 MPa as the minimum modulus value to be used for granular layer. A value of 100 MPa was adopted in the present model as a minimum modulus value.

3.3.2 Effect of Seed Modulus

The effect of initial layer modulus i.e. seed modulus on the pavement responses was examined. The response obtained from the FE model for different seed moduli is given in

Table 1. The density value considered for granular material was 2250 kg/m³. For shoulder and subgrade layer the density was taken as 2000 kg/m³. The analysis indicated that the results were not significantly influenced by the initial seed modulus values selected for the granular layer. Based on this, a seed value of resilient modulus as 250 MPa was used for granular layer elements in the present study.

Table 1. Pavement responses for different seed moduli of granular layer

Seed Modulus (MPa)	Pavement Response		
	Central Surface Deflection (mm)	Vertical Stress over Subgrade (MPa)	Vertical Strain over Subgrade
200	-0.82273	-4.93×10^{-2}	-9.98×10^{-4}
250	-0.82274	-4.93×10^{-2}	-9.98×10^{-4}
300	-0.82275	-4.93×10^{-2}	-9.98×10^{-4}
400	-0.82275	-4.93×10^{-2}	-9.98×10^{-4}

3.3.3 Nonlinearity Effect in Responses from Granular Layer

Comparison of vertical strain from the top of subgrade along the axis of symmetry of the dual wheel considering linear and nonlinear response of granular layers is shown in Fig. 6. Similarly, comparison of the surface deflection values is shown in Fig. 7.

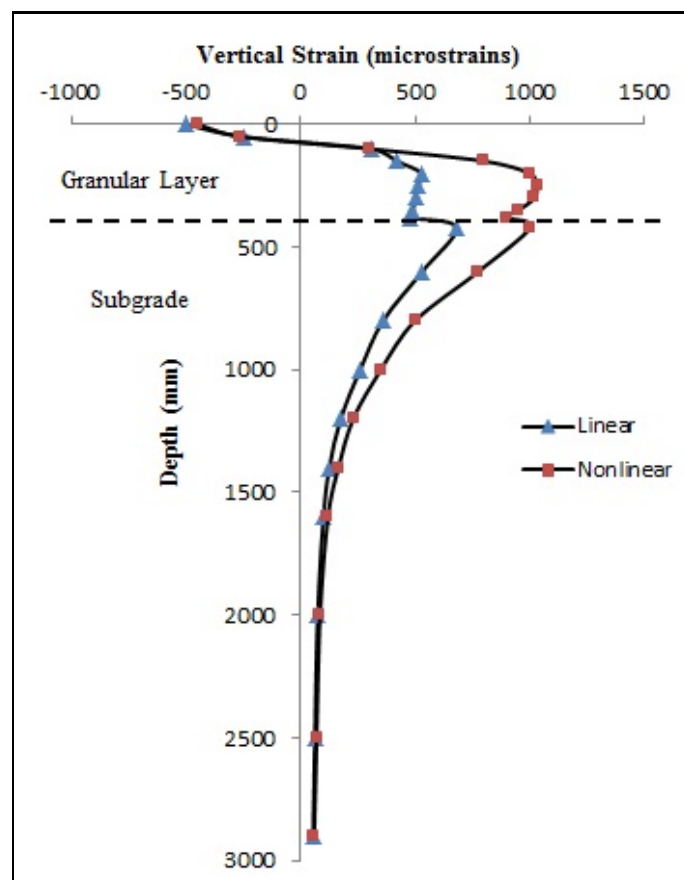


Figure 6. Effect of nonlinearity in granular layer on vertical strain

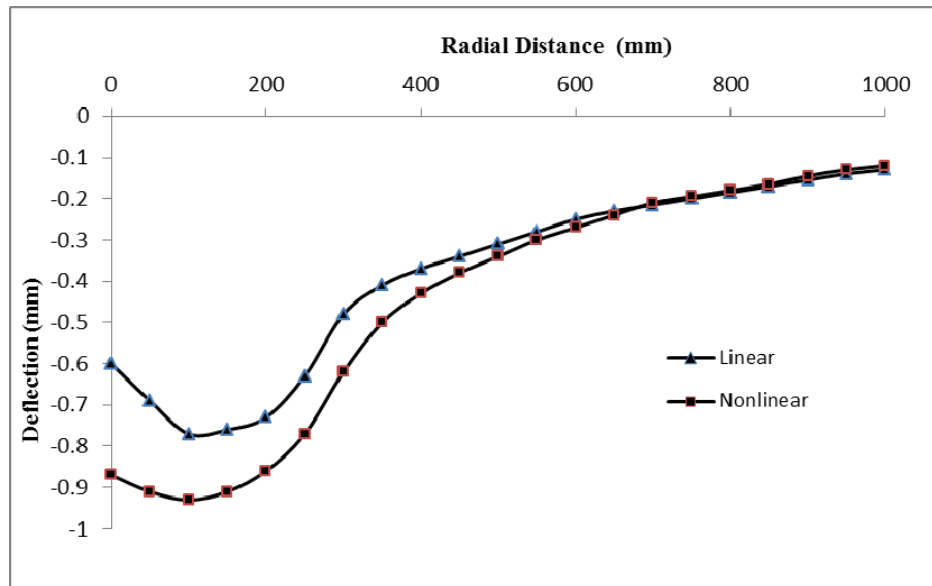


Figure 7. Effect of nonlinearity in granular layer on surface deflection

Significant difference was noted in the critical response obtained from the two approaches. The maximum difference in the vertical strain was 104% in granular layer and 54% in subgrade. Higher strain was observed near the center of the granular layer, whereas, in the case of subgrade, it occurred at the interface of the granular layer with subgrade layer. Nonlinear analysis (with respect to granular layer) resulted in similar maximum strains in the granular and subgrade layer. Vertical strain became same and constant below the depth of 1500 mm. In the case of surface deflection, the difference was observed to be 45%. It occurred at the axis of symmetry. With the increase in radial distance from the axis of symmetry, the variation in deflection estimated by the two analyses was observed to be reducing. Deflection under nonlinear analysis was found higher.

3.3.4 Pavement Parameter's Effect on Critical Responses

Subgrade moduli and granular layer thicknesses were varied to assess their effect on critical pavement responses. The variation in vertical strain, stress and deflection with granular layer thickness is shown in Fig. 8 to 10 respectively. Similar variation with subgrade strength is shown in Fig. 11 to 13. It was observed that:

- The variation in the vertical strain over the subgrade layer was found reducing sharply (for the given range of granular layer thickness) with the increase in subgrade modulus.
- For a given thickness of the granular layer, the vertical strain over the subgrade layer was found reducing with the increase in subgrade modulus.
- The variation in the vertical stress over the subgrade was found to be reducing with the increase in the granular layer thickness.
- For a given thickness of the granular layer, vertical stress over the subgrade decreased with the decrease in subgrade modulus.
- For a given subgrade modulus, vertical subgrade stress increased as the granular layer thickness decreased.
- Surface deflection was also found to be reducing with the increase in granular layer thickness for the given modulus of subgrade layer. As the modulus increased, the surface deflection became more or less constant (for 60 MPa and above).

- For the same thickness of granular layer, the surface deflection decreased drastically with the increase in subgrade modulus.
- It was observed that as the subgrade modulus increased to 100 MPa, the surface deflection became constant at 0.60 mm irrespective of the thickness of the granular layer. This has significance in terms of design of flexible pavements.

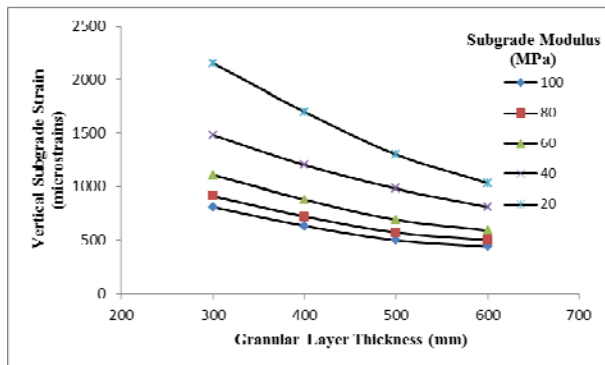


Figure 8. Effect of granular layer thickness on vertical subgrade strain

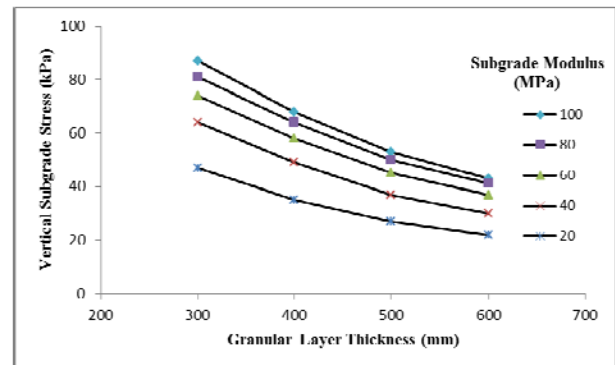


Figure 9. Effect of granular layer thickness on vertical subgrade stress

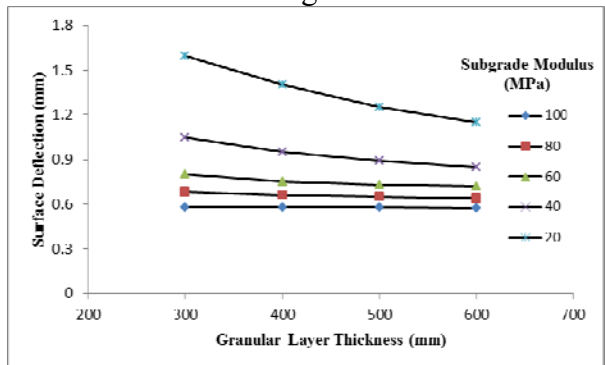


Figure 10. Effect of granular layer thickness on surface deflection

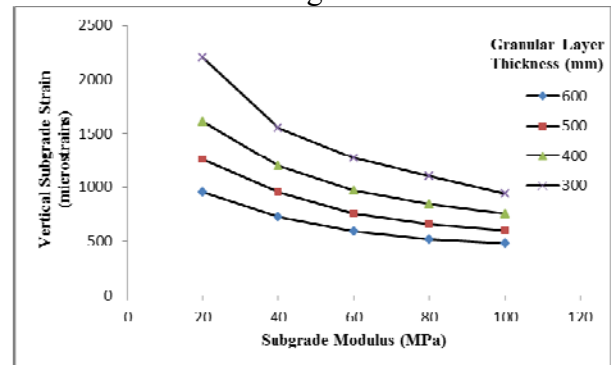


Figure 11. Effect of subgrade strength on vertical subgrade strain

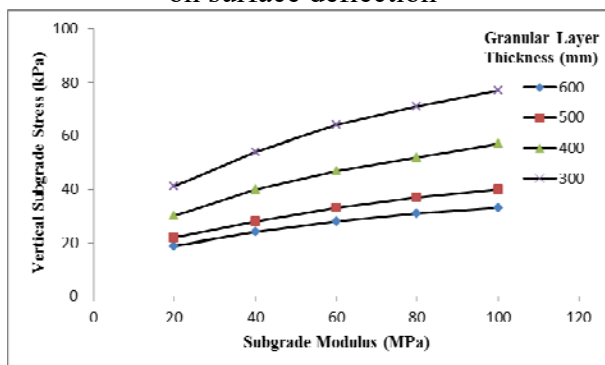


Figure 12. Effect of subgrade strength on vertical subgrade stress

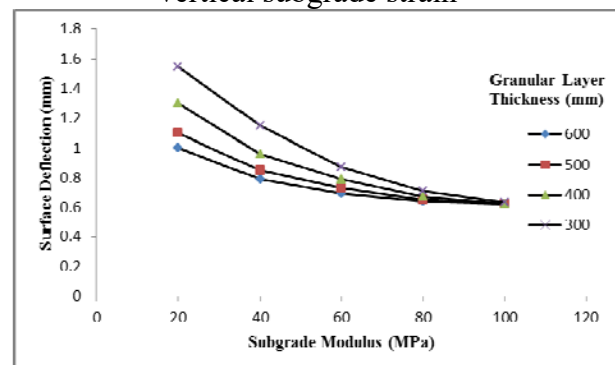


Figure 13. Effect of subgrade strength on surface deflection

4. MODELING NONLINEARITY IN BOTH THE LAYERS

In most of the FE analyses the behavior of subgrade is considered as linear-elastic. The main focus of these analyses remains on the nonlinearity of bituminous layer and granular layer. Some studies adopted nonlinearity in subgrade by using elastoplastic material models for

subgrade layer. Saad *et al.* (2005) used the elastoplastic strain hardening (modified Cam-Clay model), whereas Hossain and Wu (2002) used the elasto-perfectly plastic (Drucker-Prager) model to account for the nonlinearity in the subgrade layer. For roads sections having thick bituminous surfacing, the stress in the subgrade layer is usually small and hence considering the subgrade to be linearly elastic is not likely to have a significant effect on the response of the upper layers. But for thin bituminous surfaced pavements, granular materials constitute the main structural layer and the relatively larger stresses on subgrade cause the material to behave nonlinearly. To assess the effect of nonlinearity in the subgrade layer, the $k-\sigma_d$ model developed by Seed *et al.* (1962) was used in this study. It is given by Eq. 3.

$$M_R = 300\sigma_d^{-0.5} \quad (3)$$

where,

M_R : Resilient modulus in MPa, and
 σ_d : Deviator stress in kPa.

The effect of initial (seed) modulus on the critical pavement responses was observed by varying the seed subgrade modulus value from 30 MPa to 80 MPa. Sensitivity analysis was carried out and it was observed that variation in seed modulus of subgrade layer has insignificant effect on the pavement responses. So, subgrade seed modulus value of 50 MPa was considered. It was initially assigned to all the elements of the subgrade layer for computing the deviator stress for each element. Macros were written for this purpose. The moduli values changed in each iteration till the selected convergence criterion (as given by Eq. 2) was met.

Considering this analysis was carried out again assuming nonlinear behavior in both the layers. The pavement responses obtained from linear elastic analysis and nonlinear analysis (considering nonlinearity in both granular base and subgrade) were compared and shown in Fig. 14 and Fig.15. It was observed that nonlinear analysis yielded maximum vertical subgrade strain values that were about 59% larger than those obtained using linear analysis. The change in pattern of strain values was observed at the interface of the granular layer and subgrade. It was observed from the analysis that strain follow the asymptotic curve after the depth of 1500 mm. The variation in the strain values for linear and nonlinear analysis became negligible after the depth of around 1400 mm. At depth of 1600 mm, the value of strain observed (106×10^{-6}) was almost 10% of the highest value of strain i.e. 1050×10^{-6} . It could be inferred that subgrade depth of 1200 mm i.e. 3 times the thickness of granular layer should be considered for calculation of stresses and strains in pavement layers.

Deflection values were higher for nonlinear analysis up to radial distance of 420 mm. After that deflection due to linear analysis was observed to be higher as compared with deflection due to nonlinear analysis. Maximum value of surface deflection was observed at radial distance of 120 mm from the center of the spacing of tires. Maximum surface deflection obtained considering nonlinearity in both the layers was 37% larger than the case when linearity is considered in both the layers.

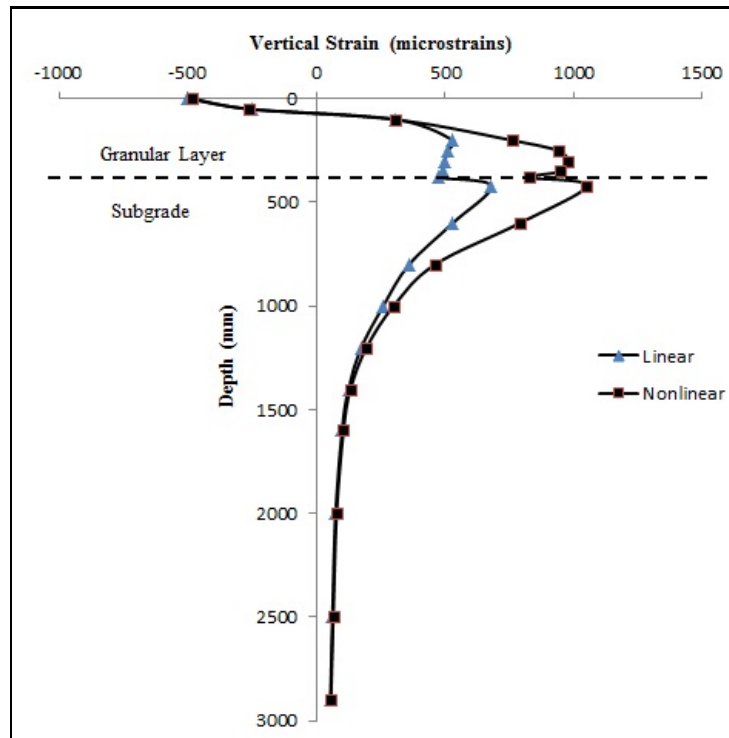


Figure 14. Vertical subgrade strain for nonlinearity in both the layers

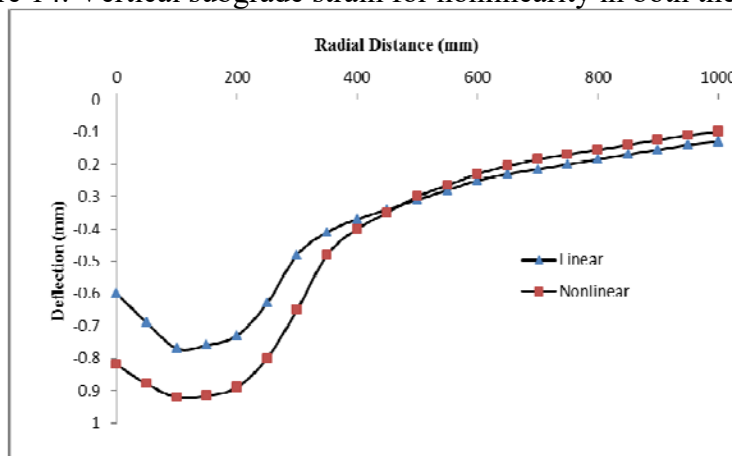


Figure 15. Surface deflection for nonlinearity in both the layers

5. CONCLUSIONS

Critical pavement responses (like vertical subgrade stress, vertical subgrade strain and surface deflections) were obtained from the 3-D FE analysis. Analysis was done considering the material to behave linearly and nonlinearly. Nonlinearity was modeled in different ways and applied to both the layers. Two combinations of these cases were analyzed and compared with pure linear behavior. Following are the important conclusions from this analysis.

- Nonlinearity was modeled in granular layer using Drucker-Prager model. Compared to linear analysis, the maximum increase was found to be 7% and 12% for surface deflection and vertical subgrade strain respectively.
- For realistic simulation of the granular materials, nonlinear stress-dependent constitutive model was used in FE analysis. Significant difference was noted in the critical responses obtained from the linear and nonlinear (granular layer) approach.

Maximum increase in vertical strain was 104% (54% in subgrade), and was up to 45% for surface deflection (at the axis of symmetry) as compared to linear analysis.

- For a given subgrade modulus, the vertical subgrade strain, vertical subgrade stress and deflection decreased with an increase in the thickness of granular layer. Similarly, for a given thickness of the granular layer, the vertical subgrade strain and surface deflection decreased, whereas, vertical subgrade stress increased with the increase in subgrade modulus. The surface deflection became constant at 0.6 mm for subgrade modulus of 100 MPa and above, irrespective of the thickness of the granular layer.
- Nonlinearity was modeled in both granular and subgrade layer and was compared with linear analysis. Critical responses increased by 37% to 59% (surface deflection and vertical subgrade strain respectively) from those obtained using linear analysis. It was observed from the analysis, that strain became asymptotic to axis after the depth of 1500 mm. At depth of 1600 mm, the value of strain observed was almost 10% of the highest value of the strain. It could be inferred that subgrade depth of 1200 mm, i.e. 3 times the thickness of the granular layer should be considered for calculation of stresses and strains in pavement layers.

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